Int. J. Heat Mass Transfer. Vol. 20. pp. 259-267. Pergamon Press 1977. Printed in Great Britain

THERMAL PROPERTY ESTIMATION UTILIZING THE LAPLACE TRANSFORM WITH APPLICATION TO ASPHALTIC PAVEMENT

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(*Received* 3 *November* 1975 *and in revised form* 7 *June* 1976)

Abstract—This paper offers a new method for measurement of thermal properties of solids which are subject to *arbitrary* heating conditions. The method utilizes the Laplace transform and can be used for many geometries, but the primary case discussed here is that of a semi-infinite, homogeneous body. The calculations are relatively simple to perform with modern hand-held calculators, and the method can be utilized for determining thermal diffusivity when only temperatures are measured and for determining both thermal conductivity and volumetric specific heat if, in addition, the surface heat flux is known.

The method is applied to the measurement of thermal properties of asphaltic pavement using temperature measurements from actual service conditions. Unlike other methods, exact solutions based on steady periodic heating conditions are not required. Transient temperature measurements in asphalt pavement are analyzed for two outdoor locations in Michigan, and calculated thermal property values of asphaltic pavement are compared with those found by using nonlinear estimation.

NOMENCLATURE

c_p, specific heat at constant pressure [J/kg-K];
k, thermal conductivity [W/m-K];
m, number of Laplace transform parameter values used in equation (10);
n, number of equal time divisions in equation (13b) and number of different experiments or thermocouples in equation (10);
q, heat flux [W/m^2];
s, Laplace transform parameter [s^{-1}];
S, sum of squares function defined by equation (10);
t, time [s];
T, temperature [K];
T', temperature increase above T_{in} [K];
T_{in}, initial temperature [K];
W_{ij}, weighting factor used in equation (10);
x, distance from the heated surface [m];
x_j, location of the jth thermocouple [m].

Greek symbols

α, thermal diffusivity, $k/\rho c_p$ [m^2/s];
θ, Laplace transform of $T'(x, t)$ [Ks^{-1}];
ρ, density [kg/m^3].

1. INTRODUCTION

THERE are many materials for which thermal properties are needed for naturally occurring heating conditions. Examples might include natural heating and cooling of soil and pavement: cryosurgery; and cooking, refrigeration, and freezing of food. In some cases a simple method of analysis for thermal properties is required. This paper presents a method that is relatively simple to use and can utilize temperatures that result from *in situ* conditions. Important restrictions are that the temperature distribution be one-dimensional and that the thermal properties be considered temperature-independent for a given set of data.

The method is general in that it can be employed for a variety of geometries, including finite plates, long cylinders, spheres, and infinite regions. In order to reduce the scope of the paper, however, only the semi-infinite homogeneous geometry is considered. To demonstrate the application of the method, data obtained from asphaltic pavement are analyzed. This serves to introduce some realistic problems, although the method has potential applications to many other situations as well.

Measurement of the thermal properties of asphaltic pavement in its *in situ* condition is important because of the unknown effects of moisture and aging under service conditions. Also, properties of asphaltic pavement fluctuate because of the pavement's highly variable composition; hence, a method of analysis that would be appropriate for field use with actual pavements may be of interest. Incidentally, this might also apply to *in situ* measurements of thermal properties of permafrost in connection with Alaskan oil pipelines. Finally, a need for thermal property data on asphaltic pavement exists because a literature search [1] revealed very limited data, particularly for thermal properties as a function of temperature and under natural conditions.

The thermal properties of interest are thermal diffusivity, thermal conductivity, and volumetric specific heat. The primary emphasis in this paper is upon thermal diffusivity, but methods for obtaining the others are also discussed.

A number of methods have been proposed for estimating thermal diffusivity from *in situ* data obtained from soil and pavement [1]. These methods frequently depend upon the assumption of steady periodic heating, however. Though these techniques provide simple means of estimating diffusivity, the variable characteristics of natural heating tend to invalidate the results of such methods. Another much more powerful procedure, sometimes called nonlinear or parameter estimation, could also be used [2, 3]. Because this method uses all the data in minimizing a sum of squares function, however, it is too complex for field use or for simple analysis. The nonlinear estimation technique is recommended whenever the greatest accuracy is required and whenever a digital computer is available. It should also be employed if there is uncertainty regarding the mathematical model. Since the presence of moisture or changing composition does introduce some uncertainty, results obtained using the proposed method are compared with those obtained using the nonlinear estimation method because the latter has a greater sensitivity to time-dependent changes.

Neither period-based methods nor the parameter estimation method can satisfy the objective of this research, which is to provide a method for estimating the thermal properties of asphaltic pavement that will satisfy the following conditions for a one-dimensional, constant thermal property body:

1. The method should permit the arbitrary heating conditions produced by nature;
2. The geometry should be that of a thermally semi-infinite body;
3. The method of analysis should be relatively simple;
4. The method should permit the simultaneous estimation of all the properties, provided the energy input is known.

2. SIMPLIFIED LAPLACE TRANSFORM METHOD

The method to be developed was used by Bellman *et al.* [4, 5] to estimate parameters in the wave equation $c^2u_{xx} = u_{tt}$, where the subscripts represent derivatives. The parameters were a_0 and a_1 in the function $c = a_0 + a_1 x$. The investigators did not consider the heat-conduction equation nor use actual experimental data, however.

Asphaltic pavement is neither homogeneous nor impervious to water. Furthermore, its thermal properties are not temperature-independent. Nevertheless, the assumptions of a semi-infinite, homogeneous, nonporous solid with temperature-independent properties are made in the analysis given below. The thermal properties obtained from both field and laboratory data demonstrate the validity of these assumptions for the data analyzed. The results are discussed further below. Accepting these assumptions, the describing partial differential equation is the simple one-dimensional heat-conduction equation:

$$\frac{\partial T'}{\partial t} = \alpha \frac{\partial^2 T'}{\partial x^2}, \tag{1}$$

where $T'(x, t)$ is defined to be the temperature difference,

$$T'(x, t) \equiv T(x, t) - T_{in} \tag{2}$$

T is temperature and T_{in} is the initial temperature, a constant value throughout the body. In (1), t is time, x is position, and α is thermal diffusivity. The surface temperature has an arbitrary time variation given by $T_0(t)$. The boundary and initial conditions are given by

$$T'(0, t) = [T(0, t) - T_{in}] = T_0'(t) \tag{3}$$

$$T'(x, 0) = [T(x, 0) - T_{in}] = 0 \tag{4}$$

$$T'(\infty, t) = [T(\infty, t) - T_{in}] = 0. \tag{5}$$

The first objective is to obtain an estimate of α given $T_0(t)$, T_{in}, and some additional temperature history for a location other than $x = 0$.

Taking the Laplace transform of equations (1), (3), (4) and (5)

$$sL(T') = \alpha \frac{d^2 L(T')}{dx^2} \tag{6}$$

$$L[T'(0, t)] = L(T_0') \tag{7a}$$

$$L[T'(\infty, t)] = 0 \tag{7b}$$

where s is the Laplace transform parameter in the integral

$$L(z) = \int_0^{\infty} z\, e^{-st}\, dt. \tag{7c}$$

Solving the problem given by (6), (7a) and (7b) yields

$$L[T'(x)] = L(T_0')\exp[-(s/\alpha)^{1/2}x] \tag{8}$$

which can be solved for α to find the simple and attractive expression,

$$\alpha = \frac{sx^2}{\ln^2\{L[T'(x)]/L(T_0')\}}. \tag{9}$$

This expression is valid for any positive real value of s. Because this fact is not obvious, it is demonstrated using Duhamel's Theorem in the Appendix.

One can use (9) directly for estimating the thermal diffusivity by selecting a reasonable value for s. It is possible, however, to pick several reasonable values of s for the same experiment [4]; a range of s values is discussed below. Various similar experiments might also be performed for estimating α. From all these data one might wish to estimate the "best" value of the thermal diffusivity, but is difficult to define the best criterion when the "observations" are the Laplace transforms of temperature differences containing errors. In this context, the "best" criterion would be the one producing a minimum variance estimator that is simultaneously relatively simple to evaluate. Unfortunately, these two conditions seem to be mutually exclusive. Since the emphasis in this paper is upon simplicity, the minimization of the sum of squares function

$$S(\alpha) = \sum_{i=1}^{m} \sum_{j=1}^{n} W_{ij}\{\alpha - s_i x_j^2/\ln^2[L_i(T_j')/L_i(T_{0,j}')]\}^2 \quad (10)$$

is suggested where W_{ij} is a weighting factor, i refers to the Laplace transform parameter s_i, and j refers to different experiments or thermocouples. If nothing is known regarding the weighting factors before analyzing the data, then all the W_{ij}'s could be set equal to unity. The more accurate the expected results are for a given s_i value relative to another value s_1, the greater W_{ij} would be compared to W_{1j}.

The thermal diffusivity is estimated from (10) by differentiating $S(\alpha)$ with respect to α, setting the result equal to zero, and solving for the estimate $\hat{\alpha}$:

$$\hat{\alpha} = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} s_i x_j^2 W_{ij}/\ln^2[L_i(T_j')/L_i(T_{0,j}')]}{\sum_{i=1}^{m} \sum_{j=1}^{n} W_{ij}}. \quad (11)$$

Hence, to estimate α it is necessary to have measurements from at least two thermocouples, one at x_0 and another at x_1; a measurement of the distance difference $x_1 - x_0$; and a reasonable value of "s". If there are measurements at only one interior location (in addition to x_0) and only one s is chosen, then $m = n = 1$ and (11) reduces to (9).

The temperature $T_0(t)$ can be completely arbitrary, and it is only necessary to evaluate the integral in the Laplace transform for the measurements at x_0 and x_i.

One important assumption in this method is that the initial temperature is uniform throughout the semi-infinite body. The lack of this uniformity necessitates the process of correcting the initial condition. Corrections for a nonuniform initial temperature can be made [3], but a uniform initial temperature can be approximated by choosing the starting time when this is true or by removing an insulating blanket just before the start of the test.

3. LAPLACE TRANSFORM CRITERIA

The function $\exp(-st)$ for various values of "s" is plotted vs st and t in Fig. 1. For a periodic temperature variation $T(x, t)$ with a constant value of T_i, the variation of $T' = \Delta T = T(x, t) - T_i$ is sinusoidal, as is shown in Fig. 2(a). It is observed that the magnitude of $\exp(-st)$ shown in Fig. 1(b), as well as the product $\Delta T \exp(-st)$ shown in Fig. 2(b), decreases on the average with increasing values of st. For $st > 6$, the contribution of T' in (11) is usually insignificant since $\exp(-6)$ is the small value of 0.0025. Therefore, real "s" values should be chosen so that they are greater than 6 divided by the maximum time, or

$$s \geqslant 6/t_{max} \quad (12)$$

where t_{max} is the maximum experiment duration. Note that this choice of s is in no way based on steady state

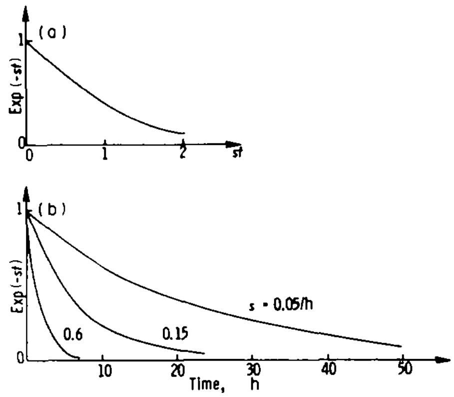

FIG. 1. Graphs of the exponential $\exp(-st)$.

conditions. If only one s value is to be used, it is recommended that it be at approximately $6/t_{max}$. Additional values up to $30/t_{max}$ are reasonable. Even larger values are theoretically possible but these values in effect use only the earliest temperature rises which have relatively low "signal to noise" ratios. See Fig. 2(b).

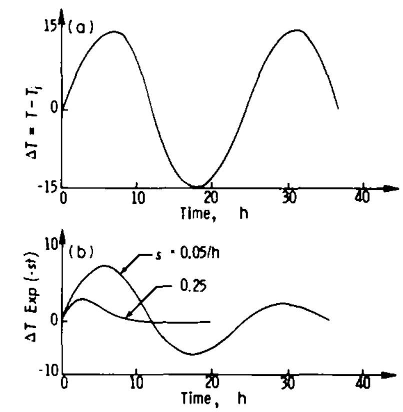

FIG. 2. Graph of $\sin \pi t/12$ and this function multiplied by $\exp(-st)$.

There are several ways to evaluate the Laplace transform of arbitrary functions. One is to use electronic integrated circuitry. Another is to evaluate the integral in the transform by using a summation, i.e. by using the trapezoidal or Simpson's rule [4]. The expression used herein to approximate the Laplace transform integral given by (7c) is a form of the trapezoidal rule given by

$$L(z) \approx \left[\sum_{i=1}^{n-1} z_i e^{-st_i} + z_0/2\right]\Delta t \quad (13a)$$

in which z is some function of t and z_i is z evaluated at t_i. The region 0 to t_{max} is divided into uniform intervals of

$$\Delta t = t_{max}/n, \; t_i = i\Delta t. \quad (13b)$$

The term at time t_{max} is omitted in (13a) because the contribution would be small. It is necessary to make n large enough to get the desired accuracy; $n = 10$ is frequently satisfactory.

It is also important to note that the Laplace transform is evaluated numerically. *No* functions are used to approximate z in (13a), rather numerical values of z at different times are utilized. In particular, z is not approximated by functions of the form $\exp(At)$, where A has a positive, real value. This is done for two reasons. First, Laplace transform pairs do not exist if the real part of s is less than A. Second, naturally occurring temperature histories do not grow exponentially with time for extended periods.

4. DESCRIPTION OF ASPHALTIC PAVEMENT

Asphaltic pavement may be considered as a system composed of solid, semisolid, or gaseous phases as well as moisture. The solid phase, also called aggregate, consists of sand, gravel, crushed stone, slag, and mineral filler. The semisolid phase is the viscoelastic asphaltic material produced from petroleum in a variety of types and grades ranging from a hard, brittle material to an almost water-thin liquid. The gaseous phase is the air which fills the voids. The aggregate is bound together by the asphaltic material, which may compose 5% by weight of the mixture.

Asphaltic pavement is usually considered to have three "courses" — wearing, binder and base. The surface is provided by the wearing course, which is a well compacted, hot-rolled asphaltic mix. The base course may have a high ratio of voids.

Water in either vapor or liquid phase enters the pavement through the voids. The presence of accessible pores, crevices, and capillary forces results in the penetration of water into the pores. The wearing course usually has a very low permeability to water, while water can easily penetrate in open-grade base mixtures.

5. EXAMPLE USING EXACT DATA

To illustrate the procedure for estimating α, consider the case of a semi-infinite body which has a step increase in temperature of T_0': the temperature history is the well-known result of

$$T'(x,t) = T_0' \operatorname{erfc}[x(4\alpha t)^{-1/2}]. \tag{14}$$

For simplicity let T_0' be equal to 1. Let there be two thermocouples measuring temperature, one at $x = 0$ and the other at x_1, where x_1 and α are so chosen that x_1^2/α is equal to 1/h.

The temperature rise T' at x_1, e^{-st}, and $T'e^{-st}$ is depicted in Fig. 3. Also, the running summation analogous to the sum inside the brackets of (13a) is given; this is for approximating the Laplace transform of the temperature at x_1. The integral for $x = 0$ is evaluated exactly as

$$T_0' \int_0^{t_{max}} e^{-st}\,dt = \frac{T_0'}{s}[1 - e^{-st_n}].$$

In each case let s be the value of 2, which means that from (12) the maximum time considered can be about 3. Notice that the summation in Fig. 3 approaches a constant at this value.

A running set of values of the thermal diffusivity α is calculated using (9), with the integrals progressively approximating the Laplace transform by increasing the time values of the upper limits. For $x = 1$ m, the α value calculated using the trapezoidal rule until $t = 3$ h and with $\Delta t = 0.25$ is 1.0027 m²/h, or 0.27% too large. Figure 3 also shows the effect of shorter times on the α value obtained. Note that the integral of the $T'(x_1, t)\exp(-st)$ function might appear to be somewhat crudely approximated with Δt as large as 0.25 because the function changes shape so greatly for small t values. However, if time steps as small as 0.05 are used, the error is only reduced to 0.09% at $t = 3$ h.

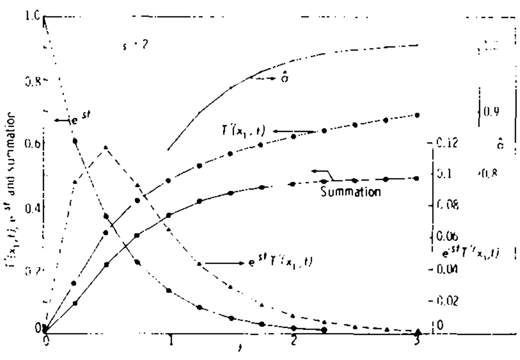

FIG. 3. Curves for example using data correct to four significant figures.

If any other s value up to 10 is used rather than 2, extremely accurate α values are still obtained. For even larger s values the accuracies tend to be poorer. This is because the approximate expression for the Laplace transform given by (13a) is inaccurate as the number of terms of $z_i\exp(-st_i)$ that are significantly different from zero approaches zero. See Fig. 2b.

It is true that the method described in (9) and (13) is more tedious to implement than a simple algebraic equation. With a modern electronic calculator such as the Hewlett–Packard 35, however, the calculations can be performed in just a few minutes. If a programmable calculator is available, the solution can be obtained in about the time that it takes to input the measured temperatures. In either case, evaluation of the thermal diffusivity can be achieved without utilizing a digital computer.

6. HEAT FLUX CONDITION

If the heat flux is known as a function of time at $x = 0$, not only can the thermal diffusivity be found, but the thermal conductivity, k, and specific heat, c_p, can also be found. Suppose the heat flux $q(t)$ is known,

$$q(t) = -k\left.\frac{\partial T}{\partial x}\right|_{x=0} \tag{15}$$

or, more precisely, assume that the Laplace transform of $q(t)$ is known,

$$L[q(s)] = \int_0^{\infty} e^{-st}q(t)\,dt. \tag{16}$$

Taking the Laplace transform of (15) results in

$$-k\frac{\partial\theta}{\partial x}\bigg|_{x=0} = L[q(t)],\quad \theta = L[T'(x)]. \tag{17}$$

Using (8) in (17) yields

$$k\rho c_p = \{L[q(t)]/\theta_0\}^2(1/s),\quad \theta_0 = L(T_0'). \tag{18}$$

Thus, if information regarding the temperature and heat flux in a semi-infinite body is available, the values of $k\rho c_p$ and α can be measured. If α and $k\rho c_p$ are measured, the values of k and ρc_p can be obtained directly from

$$k = [\alpha(k\rho c_p)]^{1/2} \tag{19}$$

$$\rho c_p = [k\rho c_p/\alpha]^{1/2}. \tag{20}$$

provided the density ρ is also known.

7. EXPERIMENTAL RESULTS

7.1. *Laboratory data*

Several sets of experimental data were analyzed. One set was from laboratory tests on a cylindrical core specimen 7.62 cm in diameter and 3.81 cm thick. This specimen was prepared so as to be similar to the wearing and binding courses of the asphaltic pavement in the field tests. Two thermocouples were embedded in each of four planes below the heated surface. Tests of two minutes duration were run with the specimen at different initial temperatures from 239 to 322 K (−30 to 120°F).

The specimen was heated (or cooled) by using a hydraulic system [6] to bring a 7.62 cm dia copper calorimeter into good contact with it. The calorimeter was initially at a different temperature from the specimen. In Fig. 4, the calorimeter surface temperature and the specimen temperature histories are shown for respective locations of 0.64, 1.27, 2.54 and 3.18 cm from the heated surface. Considering the inhomogeneous composition of asphaltic pavement, it is remarkable that the thermocouple responses at each depth were as close as those shown in Fig. 4. Evidently the assumption of a homogeneous solid was reasonable in connection with the heat transfer.

The temperatures in the specimen and in the calorimeter were digitized from thermocouple signals using an IBM 1800 computer. The transient temperatures in the calorimeter can be used to calculate the heat flux at its surface [7]. Even if there is a resistance to heat flow at the interface, the heat flux leaving the calorimeter must be the same as that entering the specimen at the common interface.

The data from these laboratory tests have been analyzed in two different ways. The first is that detailed by this paper, i.e. computer calculations based on (9), (13), (18), (19), and (20). The second is the nonlinear estimation method.

A visual comparison of the two methods can be obtained through an examination of the results shown in Table 1. Note that the different methods give very similar values, with the differences being less than about ±5%. The average temperatures given in Table 1 are the simple averages of highest and lowest temperatures measured in the cases of interest.

Table 1. Comparison of thermal diffusivity of asphaltic pavement calculated from laboratory data using the Laplace transform and nonlinear estimation methods

Case	Condition	Average temp., (K)	Thermal diffusivity × 10^6, (m^2/s) Laplace T. method	Nonlinear est.
1.1	dry	323	1.01	1.03
1.2	dry	323	1.08	1.06
1.3	dry	323	1.01	1.08
1.4	dry	323	0.98	1.06
2	dry	318.4	1.06	1.16
3	dry	312	1.08	1.11
4	dry	308.7	1.32	1.26
5.1	dry	279	1.24	1.21
5.2	wet	276.7	1.65	1.57
5.3	dry	278.7	1.29	1.42
6.1	dry	255.6	1.39	1.34
6.2	wet	255.3	1.83	1.73
6.3	dry	260.0	1.60	1.50

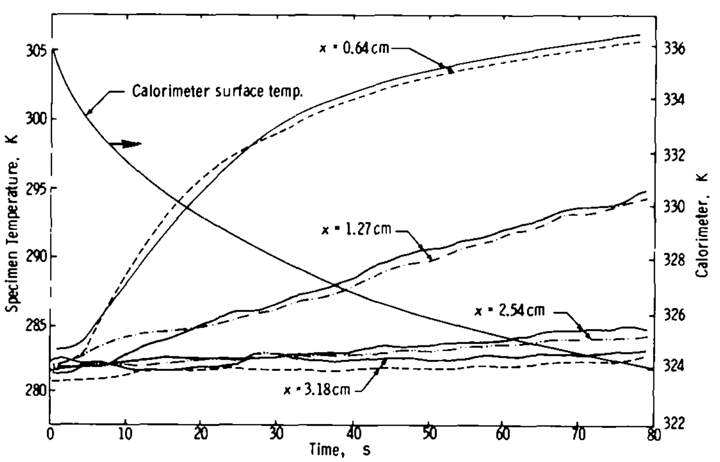

Fig. 4. Measured temperature at the calorimeter surface and various depths of the laboratory specimen (solid and dashed lines refer to first and second thermocouple sets).

Another comparison is suggested by the average values given in the first two rows in Table 2, where thermal diffusivity results for the dry specimens at 255 and 311 K are presented. These α values were obtained using least squares, with α assumed linear in temperature. The data used came from Table 1. At 255 K the Laplace method is 3.2% higher than the nonlinear estimation method: at 311 K it is 2.4% lower.

Table 2. Summary of results for dry asphaltic pavement, wearing and binding courses

Source of data	Method of analysis	Thermal diffusivity $\times 10^6$, (m^2/s) α at 255 K (0°F)	α at 311 K (100°F)
Laboratory	Nonlinear est.	1.44	1.15
Laboratory	Laplace trans.	1.48	1.12
Field	Nonlinear est.	1.51	1.28
Field	Laplace trans.	1.56	1.20

It is significant that results of the two methods agree so closely. The nonlinear estimation method provides a means of checking the model because differences of the calculated and measured values of temperature can be investigated for systematic deviations from test to test. The presence of such systematic effects would indicate an inadequacy in the model due to the presence of moisture, nonhomogeneity, etc. None was noted and thus the simple heat-conduction model given by (1) appears to be adequate for the conditions tested.

7.2. *Field data*

In situ temperature measurements made by other investigators were also analyzed. One set of data for 18 cm thick asphaltic pavement, came from Gratiot County, Michigan [8], and another set of data came from Bishop Airport, Flint, Michigan, [9] where the pavement is 48 cm thick. Figure 5 depicts some typical results for the airport location on a sunny day. Temperature histories are shown as deep as 107 cm in the underlying soil.

In analyzing data such as that shown in Fig. 5, the initial time should be assumed to be when the temperature is relatively uniform, yet is followed by rapid changes in temperature. These conditions are satisfied at about 8.00 a.m. Because the temperature distribution is not uniform at any time, however, a correction for this condition should be applied [3].

Values of the thermal diffusivity calculated for data from the two locations are shown in Fig. 6. Data were used only for days having no precipitation. Moreover, only data corresponding to the upper pavement levels (i.e. the wearing and binding courses) are included. The two methods of analysis (nonlinear estimation and

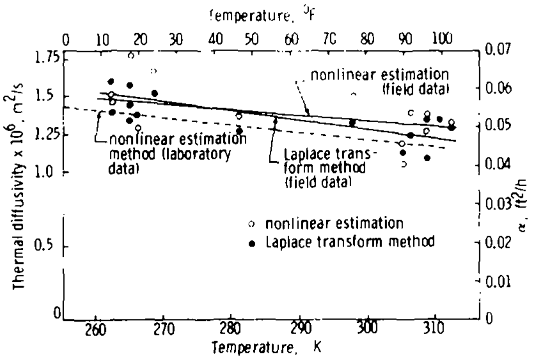

FIG. 6. Calculated thermal diffusivity of asphaltic concrete from field data using two methods.

Laplace transform) were used. Least squares lines through the data are shown, along with a dashed line which denoted the laboratory data analyzed using nonlinear estimation. These lines are described in Table 2 by the values at 255 and 311 K. There are some differences in the α values due to the method of the calculation and the source of the data. The field data, for example, is as much as 8% larger than the laboratory data. (This could be due to the presence of moisture or the effects of aging.) The differences in the average values displayed in Table 2 for a given temperature are quite small, however, compared to the range of the individual α values given in Table 1 or in Fig. 6. For example, in Table 2 at 255 K there is

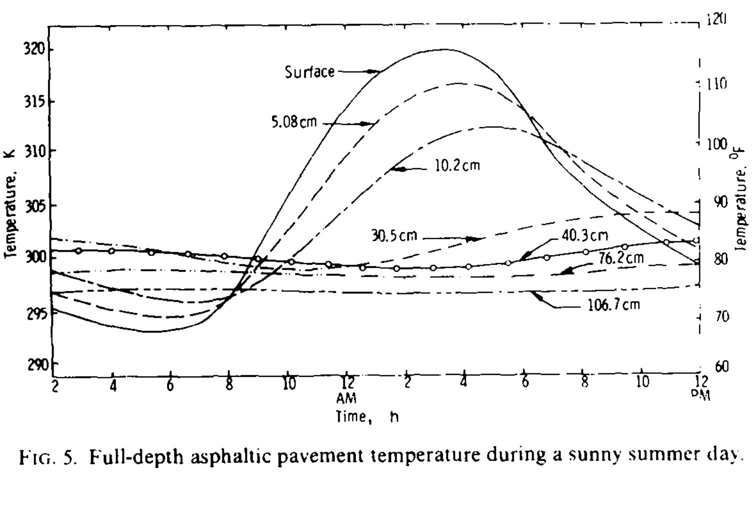

FIG. 5. Full-depth asphaltic pavement temperature during a sunny summer day.

about a $\pm 16\%$ variation near 265 K. Hence, from these variations and also from an inspection of Fig. 6, any set of values given in a row in Table 2 might be used. Since the laboratory data are more consistent than the field data and since the nonlinear estimation values should be more accurate, the recommended α values are those given by the dashed line in Fig. 6, which is described by the first row of Table 2. For convenience, these values are repeated in the recommended values in Table 3.

Table 3. Recommended thermal properties for dry asphaltic pavement

	Temperature	
	255 K (0°F)	311 K (100°F)
α, m^2/s	1.44×10^{-6}	1.15×10^{-6}
α, ft^2/h	0.056	0.045
k, $W/m \cdot K$	2.88	2.28
k, $Btu/h\,ft \cdot F$	1.66	1.31
ρc_p, $J/m^3 \cdot K$	2.00×10^6	1.97×10^6
ρc_p, $Btu/ft^3 \cdot F$	29.8	29.4

The field data were also analyzed to obtain average thermal diffusivity values representing all the courses, i.e. wearing, binding, and base. On the average, the values were only about 3% less than the corresponding values for just the upper two courses. This is a negligible difference.

In both the field and laboratory data the "wet" pavement gave consistently higher values. For the laboratory data both tests gave close to 20% increase, while the field data had about 15% increase. Based on the limited data at hand, it seems that a 20% increase over the dry values is indicated. It is felt that this increase is mainly a result of filling the voids in the pavement, thereby reducing the resistance to heat flow. Since the wearing course, in particular, is well-compacted, the migration of moisture would be slow and thus would not be the dominant mode for the increase in heat transfer. The increase is probably due to greater conduction resulting from water rather than air being in the voids.

8. RECOMMENDED VALUES FOR DRY ASPHALTIC PAVEMENT

Based on the results discussed above, some recommended values of thermal diffusivity, thermal conductivity, and a density- specific heat product are given in Table 3. These values are for the wearing and binding courses of dry asphaltic pavement. The recommended values of α and k are within $\pm 20\%$ of most of the data. The ρc_p product agrees to $\pm 8\%$ of all the data. These results are appropriate for the materials investigated, but due to the variability of the natural components different values might be found for other cases. Hence a rapid method for the measurement of properties may be needed.

It is important to point out that the values of α and k are considerably higher (by a factor of almost two) than those used by some other investigators. If the low values are used, calculations of frost depth will indicate considerably less penetration than is actually present, if the true property values are actually the larger values contained herein.

9. COMPARISON WITH LITERATURE VALUES

There is relatively good agreement in the literature on the specific heat and density of asphaltic pavement. The values used in [10, 11] are $c_p = 921\,J/kg \cdot K$ and $\rho = 2242\,kg/m^3$, which has a ρc_p product of $2.07 \times 10^6\,J/m^3 \cdot K$. This is only 5% less than the 311 K (100°F) value given in Table 3.

The reported thermal conductivity values are considerably more variable than ρ and c_p. In [10, 11] a value of thermal conductivity of 1.2 W/m · K was used. This was primarily intended to be a typical value valid for application of hot-mix layers starting at about 422 K and then cooling to 353 K. Though the value for k given in Table 3 is 2.28 W/m-K at 311 K, it decreases to 1.2 at 408 K (if linear extrapolation is permitted). Many other references could be cited, including Aldrich [9], who used a value of 1.5 W/m · K; Saal [12], who gave 2.23 W/m · K; and O'Blenis [13], whose values varied from 0.85 to 2.32 W/m · K.

Given the above values of thermal conductivity and ρc_p, the thermal diffusivity can be calculated and considerable variation can again be expected. Corlew and Dickson [10, 11] used a value for α of $5.86 \times 10^{-7}\,m^2/s$, which is about half of that recommended herein for 311 K. They also show reasonable agreement between some experimental temperature measurements and calculated temperatures using this value. In [10], most of the comparisons show temperatures above 339 K. Using the Laplace transform with their data also corroborates their α value.

Consideration of the components of asphaltic pavement suggests that the variation in the above values may not be unreasonable. For example, one possible type of rock to be used in the aggregate is limestone; in units of W/m · K the thermal conductivity is reported to be 2.1 at 273 K [14], 0.7 at 294 K [15], and 1.2 at 372 K [16]. The corresponding values for the first two references of the thermal diffusivity are 4.4×10^{-7} and $8.3 \times 10^{-7}\,m^2/s$. The thermal conductivity of granite, another possible component of the aggregate, is given by 2.8 at 273 K in [14], between 1.7 and 4.0 in [17] where no temperature is given, and between 3.1 and 4.2 by Gebhart [16] (units of W/m · K are used in each case). It is realistic to assume that the disparate values cited are, to a large degree, due to the variability of the materials themselves.

Literature values for the asphalt's conductivity also vary from 0.16 to 0.76 W/m · K [12]. The latter value is 20% larger than that of water at 311 K.

10. SUMMARY AND CONCLUSIONS

A new method for measuring thermal properties is derived and illustrated through the use of analytical and experimental data. The method is straightforward, uses a minimum number of assumptions, is applicable to semi-infinite solids, and can be applied to *in situ* data. Both the thermal conductivity and the volumetric specific heat can be found if the Laplace transform of the surface heat flux can be evaluated.

New thermal property values for asphaltic pavement are given. Three different sets of data were analyzed two from *in situ* data in Michigan and one from laboratory data and recommended thermal property values for asphaltic pavement are given based on these data. Other literature values exist for comparable values of thermal conductivity and thermal diffusivity, although most literature values tend to be lower. It is suggested that the scatter in the literature values may be due in part to the variable composition of the materials, as well as to the variations in natural materials themselves.

Acknowledgement The junior author appreciated partial support from the National Science Foundation under Grant No. GK16526. Comments of Professor C. R. St. Clair, Jr. are also appreciated.

REFERENCES

1. L. K. Moulton, Prediction of the depth of frost penetration in West Virginia for pavement design purposes, Ph.D. dissertation, West Virginia University (1968).
2. J. V. Beck, Transient determination of thermal properties, *Nucl. Engng Design* **3**, 373–381 (1966).
3. A. Kavianipour, Analytical estimation of thermal properties and variation of temperature in asphaltic pavements, Ph.D. dissertation, Dept. of Civil Engineering, Michigan State University (1971).
4. R. Bellman, D. Detchmendy, H. Kagiwada and R. Kalaba, On the identification of systems and the unscrambling of data—III. One-dimensional wave and diffusion processes; Memorandum RM-5173-PR, The Rand Corp. (1966).
5. H. H. Kagiwada, *System Identification Methods and Applications*, pp. 123–142. Addison-Wesley, Reading, Mass. (1974).
6. J. V. Beck and S. Al-Araji, Investigation of a new simple transient method of thermal property measurement, *J. Heat Transfer* **96C**, 59–64 (1974).
7. J. V. Beck, Nonlinear estimation applied to the nonlinear inverse heat conduction problem, *Int. J. Heat Mass Transfer* **13**, 703–716 (1970).
8. G. P. Manz, Study of temperature variation in hot mix asphalt base, Surface course and subgrade, *Highway Res. Rec. Bull.* 150 (1956).
9. H. P. Aldrich, Jr., Frost penetration below highway and airport pavements, *Highway Res. Rec. Bull.* 135 (1956).
10. J. S. Corlew and P. F. Dickson, Methods for calculating temperature profiles of hot-mix asphalt concrete as related to the construction of asphalt pavements, *Proc. Assoc. Asphalt Paving Technol.* **37**, 101–140 (1968).
11. J. S. Corlew and P. F. Dickson, Cold-weather paving of thin lifts of hot-mixed asphalt on preheated asphalt base, *Highway Res. Rec.* 385 (1972).
12. R. N. J. Saul, Physical properties of asphaltic bitumen, in *The Properties of Asphaltic Bitumen*, edited by J. P. H. Pfeiffer, Chapter 2. Elsevier, New York (1960).
13. J. D. O'Blenis, Thermal properties of West Virginia highway materials, Master of Science Thesis, Civil Engineering Department, West Virginia University.
14. L. R. Ingersoll, O. J. Zobel and A. C. Ingersoll, *Heat Conduction with Engineering, Geological and Other Applications*, p. 288. The University of Wisconsin Press, 1 (1956).
15. F. Kreith, *Principles of Heat Transfer*, 3rd edn. Intext Educational, New York (1973).
16. B. Gebhart, *Heat Transfer*, 2nd edn. McGraw-Hill, New York (1971).
17. W. M. Rohsenow and J. P. Hartnett, *Handbook of Heat Transfer*. McGraw-Hill, New York (1973).
18. G. E. Myers, *Analytical Methods in Conduction Heat Transfer*. McGraw-Hill, New York (1971).

APPENDIX

Verification of the Independence of Equation (9) of the s Value

This Appendix is given because it is not obvious that (9) is independent of the s value chosen. For an arbitrary temperature history at the surface of a homogeneous, constant property semi-infinite body, the temperature rise at a point x is given by Duhamel's Theorem [18] as

$$T'(x,t) = \int_0^t T_0'(\lambda)\frac{\partial\phi(x,t-\lambda)}{\partial t}\,\mathrm{d}\lambda \tag{A.1}$$

where $T_0'(t)$ is the given surface temperature and $\phi(x,t)$ is the temperature response at x due to a unit step surface temperature rise.

Taking the Laplace transform of (A.1) gives

$$L[T'(x,t)] = L(T_0')L[\partial\phi(x,t)/\partial t]. \tag{A.2}$$

The derivative $\partial\phi/\partial t$ is the derivative of (14) with T_0' set equal to unity; the result is

$$\frac{\partial\phi(x,t)}{\partial t} = \frac{x}{t(4\pi\alpha t)^{1/2}}\exp\left(-\frac{x^2}{4\alpha t}\right) \tag{A.3}$$

which has the Laplace transform of

$$L[\partial\phi(x,t)/\partial t] = \exp[-x(s/\alpha)^{1/2}] \tag{A.4}$$

(see p. 446 of [18]). Then evaluating the denominator of (9) gives

$$\ln^2\frac{L[T_1'(x)]}{L(T_0')} = \left\{\ln\frac{L(T_0')\exp[-x(s/\alpha)^{1/2}]}{L(T_0')}\right\}^2 \tag{A.5}$$

$$= [-x(s/\alpha)^{1/2}]^2 = x^2 s/\alpha \tag{A.6}$$

which when introduced into (9) yields α. Notice that the s and x^2 values cancel in (9).

ESTIMATION DES PROPRIETES THERMIQUES PAR LA TRANSFORMATION DE LAPLACE, AVEC APPLICATION AUX REVETEMENTS ASPHALTIQUES

Résumé—L'article propose une nouvelle méthode de mesure des propriétés thermiques de solides soumis à des conditions de chauffage *arbitraires*. La méthode s'appuie sur la transformation de Laplace et peut être utilisée pour différentes géométries, mais le cas étudié ici est celui d'un solide semi-infini et homogène. Les calculs sont suffisamment simples pour être faits sur des calculatrices de poche et la méthode peut être utilisée pour déterminer la diffusivité thermique quand les températures sont mesurées et aussi à la fois la conductivité thermique et la chaleur spécifique quand, en plus, est connu le flux thermique à la surface.

Le méthode est appliquée à la détermination des propriétés thermiques des revêtements asphaltiques par la mesure de température dans les conditions réelles d'utilisation. Contrairement à d'autres méthodes,

des solutions exactes basées sur des conditions de chauffage périodiques et établies ne sont pas nécessaires. On analyse des mesures de températures transitoires dans un revêtement asphaltique pour deux situations extérieures en Michigan et les propriétés thermiques calculées sont comparées à celles trouvées par estimation non linéaire.

DIE ABSCHÄTZUNG THERMISCHER STOFFEIGENSCHAFTEN UNTER VERWENDUNG DER LAPLACE-TRANSFORMATION MIT SPEZIELLER ANWENDUNG AUF ASPHALT-STRASSENBELÄGE

Zusammenfassung - Es wird eine neue Methode zur Messung der thermischen Stoffeigenschaften von Festkörpern, welche beliebigen Beheizungsbedingungen unterworfen sind, vorgeschlagen. Die Methode verwendet die Laplace-Transformation und kann auf viele Geometrien angewandt werden; der wichtigste, hier diskutierte Fall ist der des halbunendlichen, homogenen Körpers. Die Berechnungen sind relativ einfach mit modernen Taschencomputern durchzuführen. Die Methode kann zur Bestimmung der Temperaturleitzahl verwendet werden, wenn nur Temperaturen gemessen werden; ist zusätzlich der Oberflächenwärmestrom bekannt, dann können auch die Wärmeleitfähigkeit und die isochore spezifische Wärmekapazität bestimmt werden. Unter Verwendung von Temperaturmessungen unter tatsächlichen Betriebsbedingungen wird die Methode zur Messung thermischer Stoffwerte von Asphalt-Straßenbelägen angewandt. Im Gegensatz zu anderen Methoden sind dabei keine exakten Lösungen aufgrund stationärer, periodischer Heizbedingungen erforderlich. Instationäre Temperaturmessungen in Asphalt-Straßenbelägen in zwei Orten in Michigan werden ausgewertet; die berechneten thermischen Stoffwerte werden mit denjenigen verglichen, die mit Hilfe nichtlinearer Abschätzungsverfahren ermittelt worden sind.

ИСПОЛЬЗОВАНИЕ ПРЕОБРАЗОВАНИЯ ЛАПЛАСА ДЛЯ ОЦЕНКИ ТЕПЛОФИЗИЧЕСКИХ СВОЙСТВ АСФАЛЬТА

Аннотация — Предложен новый метод измерения тепловых свойств твердых тел, не зависящий от условий нагрева исследуемого тела. Методика определения этих свойств базируется на использовании преобразованного по Лапласу уравнения теплопроводности для полубесконечного однородного тела. Расчетная формула относительно проста и расчет свойств может быть проведен ручными вычислительными средствами; этот метод может быть использован для определения температуропроводности, если известна температура, и для определения теплопроводности и теплоемкости при постоянном объеме, если известен тепловой поток. Метод применяется для измерения теплофизических свойств асфальта в зависимости от температуры. В отличие от других методов в данном случае не требуется иметь точные решения уравнения теплопроводности при условиях стационарного периодического нагрева. Анализируются свойства асфальтов двух районов Мичигана, полученные нестационарным методом; расчетные значения этих свойств сравниваются с данными, полученными на основе нелинейной методики.